

THERMAL STABILITY OF A REACTIVE LIQUID FLOW

T. A. Bodnar'

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Stability analysis of the solution of the system of differential partial equations describing the thermal state of a reactive liquid flow is based on reducing the infinite-dimensional problem to a finite-dimensional space containing the part of the solution that determines its stability. Within the framework of the projection method, the zero space of the corresponding generating operator is used as the finite-dimensional space. Generally, the zero space of the generating operator of the problem considered consists of its eigenfunctions. Analysis is performed for a combination of liquid-flow parameters such that the generating operator degenerates and to construct its zero space, it is necessary to use vectors generated by the Jordan chain. Calculation results are presented.

The thermal stability of tangential and axial flows of a reactive liquids has been studied previously [1, 2]. The stability limits of the solutions of the corresponding problems were found to be hypersurfaces in the space of liquid-flow physical parameters, which are treated as independent coordinates. (If just one flow parameter is retained as a bifurcation parameter and the remaining parameters are fixed, the hypersurface degenerates into a point.)

Determination of the stability of solutions of infinite-dimensional problems described by differential partial equations involves reduction of their dimension. Within the framework of the projection method [3], reduction is achieved by projecting the problem considered onto the eigenspace generated by the corresponding generating operator. In [1, 2], a thermal stability analysis was performed for combinations of liquid-flow parameters for which the generating operators have a simple structure, i.e., the algebraic and geometric multiplicities of the eigenvalues of each of the operators coincide. The zero space of a generating operator with a simple structure consists of its eigenfunctions. At the same time, for particular combinations of liquid-flow parameters, the eigenvalues of the generating operators have algebraic multiplicity exceeding the geometric multiplicity and the operators do not have a simple structure. In this case, to construct the zero space of the generating operator and to study the stability of a solution of the problem, it is necessary to use the concept of the generalized functions generated by the Jordan chain. Apparently, the necessity of determining the functional eigenspace arises only when the multiple eigenvalue is equal to zero and the real parts of the remaining eigenvalues are negative. Otherwise, the stability problem is solved at once and no additional analysis is required.

We consider the problem of the evolution of the thermal state of an axial flow of a reactive liquid in a circular channel of finite length, which is a mathematical model of a polytropic chemical reactor [4], written in functional form [2]:

$$\begin{aligned} \frac{\partial \mathbf{U}}{\partial t} &= A\mathbf{U} + B(\mathbf{U}, \mathbf{U}) + C(\mathbf{U}, \mathbf{U}, \mathbf{U}) + \Phi(0, 0) + O(|\mathbf{U}|^4), \\ \mathbf{U}(x, t_0) &= 0, \quad \mathbf{U}(0, t) = 0, \quad \frac{\partial \mathbf{U}(l, t)}{\partial x} - 0.5w\mathbf{U}(l, t) = 0. \end{aligned} \tag{1}$$

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Here x is the coordinate along the flow axis, t is time, t_0 is the initial time, w is the liquid flow velocity, and l is the length of the reactor. In the problem (1), the unknown is the vector U , which generally describes the physicochemical state of the liquid flow. The vector U is acted upon by the constant $\Phi(0, 0)$, linear (AU) , and nonlinear $[B(U, U)$ and $C(U, U, U)]$ operators, which transform it to other vectors whose dimension is identical to that of U . The dimension of the vector U is equal to the number of independent parameters describing the thermal and chemical state of the liquid. In [2], as such parameters, we used the temperature U_1 and concentration U_2 of one of the reacting liquid components. The heat source function φ and the heat loss function φ_1 have the form

$$\begin{aligned}\varphi(U_1, U_2) &= c_0 k_1 (1 + U_2 \exp(-0.5wx))(1 + k_2 c_0 (1 + U_2 \exp(-0.5wx)))^{-2} \\ &\quad \times \exp(U_1 \exp(-0.5wx)(1 + \beta U_1 \exp(-0.5wx))^{-1}), \\ \varphi_1 &= \alpha_1 U_1 \exp(-0.5wx),\end{aligned}$$

where c_0 , k_1 , k_2 , α_1 , and β are constants. The variables and parameters included in the problem (1) are dimensionless.

In this formulation, the vector U and the operators $\Phi(0, 0)$, AU , $B(U, U)$, and $C(U, U, U)$ are written as

$$\begin{aligned}U &= \left\| \begin{array}{c} U_1 \\ U_2 \end{array} \right\|, \quad \Phi(0, 0) = \left\| \begin{array}{c} \varphi(0, 0) \\ -\alpha\varphi(0, 0) \end{array} \right\|, \\ AU &= \left\| \begin{array}{cc} \partial^2/\partial x^2 + \varphi_{1,0} - \alpha_1 - 0.25w^2 & \varphi_{0,1} \\ -\alpha\varphi_{1,0} & \partial^2/\partial x^2 - \alpha\varphi_{0,1} - 0.25w^2 \end{array} \right\| \left\| \begin{array}{c} U_1 \\ U_2 \end{array} \right\|, \\ B(U, U) &= \left\| \begin{array}{ccc} \varphi_{2,0} & \varphi_{1,1} & \varphi_{0,2} \\ -\alpha\varphi_{2,0} & -\alpha\varphi_{1,1} & -\alpha\varphi_{0,2} \end{array} \right\| \left\| \begin{array}{ccc} U_1^2 & U_1 U_2 & U_2^2 \end{array} \right\|^t, \\ C(U, U, U) &= \left\| \begin{array}{cccc} \varphi_{3,0} & \varphi_{2,1} & \varphi_{1,2} & \varphi_{0,3} \\ -\alpha\varphi_{3,0} & -\alpha\varphi_{2,1} & -\alpha\varphi_{1,2} & -\alpha\varphi_{0,3} \end{array} \right\| \left\| \begin{array}{cccc} U_1^3 & U_1^2 U_2 & U_1 U_2^2 & U_2^3 \end{array} \right\|^t.\end{aligned}$$

Here

$$\varphi_{i,j} = \frac{1}{i!j!} \exp(0.5wx) \frac{\partial^{i+j}\varphi(0,0)}{\partial U_1^i \partial U_2^j},$$

α is a constant, and $\| \cdot \| ^t$ is a transposed matrix.

The linear operator AU is chosen as the generating operator. Then, to study the stability of solutions of system (1), we should solve the spectral problem $AU = 0$. The eigenvalues σ_n ($n = 1, 2, \dots$) of the operator A satisfy the quadratic equations [2]

$$\begin{aligned}\sigma_n^2 + \sigma_n(2\lambda_n^2 + 0.5w^2 + \alpha\varphi_{0,1} + \alpha_1 - \varphi_{1,0}) + \alpha\alpha_1\varphi_{0,1} \\ + (\lambda_n^2 + 0.25w^2)(\lambda_n^2 + 0.25w^2 + \alpha\varphi_{0,1} + \alpha_1 - \varphi_{1,0}) = 0,\end{aligned}\quad (2)$$

where λ_n are positive roots of the equation $\tan(\lambda l) = 2\lambda w^{-1}$, arranged in ascending order: $\lambda_1 < \lambda_2 < \dots$.

The coefficient at the higher-order exponent of Eq. (2) is equal to unity, and the remaining coefficients are analytic functions of the parameters of the problem w , α , α_1 , $\varphi_{1,0}$, and $\varphi_{0,1}$ (this corresponds to the conditions of Theorems XII.1 and XII.2 extended to the multiparameter case [5]). Hence it follows that in the space of the parameters of the problem, which are treated as independent coordinates, the stability limit of the solution of the linear equation $AU = 0$ is a smooth hypersurface, at each point of which the combination of values of the coordinates is such that the maximum real root of the first of Eqs. (2) vanishes or, if the roots are complex conjugate, their real parts vanish.

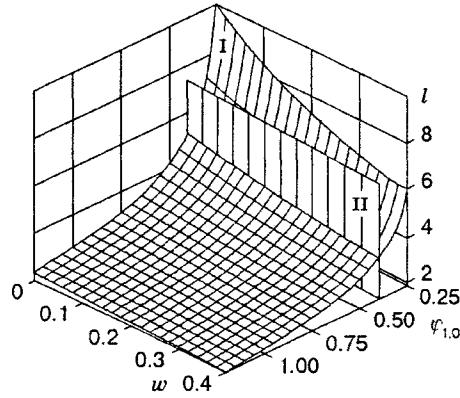


Fig. 1

A stability analysis of the bifurcation solution of system (1) for the complex conjugate and real roots of the secular equation (2) for $n = 1$ is presented in [1, 2].

It is of interest to study the stability of the solutions of system (1) at points on the hypersurface at which

$$\varphi_{1,0} - a = 0 \quad (a = 2\lambda_1^2 + 0.5w^2 + \alpha\varphi_{0,1} + \alpha_1), \quad \alpha\alpha_1\varphi_{0,1} - (\lambda_1^2 + 0.25w^2)^2 = 0 \quad (3)$$

and both roots of the first of Eqs. (2) have real and imaginary parts equal to zero: $\text{Re}\sigma_{11} = \text{Re}\sigma_{12} = \text{Im}\sigma_{11} = \text{Im}\sigma_{12} = 0$. The solution of the first of Eqs. (2) obtained for fixed values $\alpha = 1$, $\alpha_1 = 0.1$, and $\varphi_{0,1} = 0.1$ taking into account relations (3) is shown in Fig. 1 in the form of a curve of $l(w, \varphi_{1,0})$. The roots of Eq. (2) have a zero real part on surface I and a zero imaginary part on surface II and are equal to zero on the line of intersection of surfaces I and II. Surface I (except for the line of intersection with surface II) is the upper bound of the region of stable solutions of the equation $AU = 0$. The physical meaning of the calculation results is that the critical length of the reactor l decreases with increase in the coefficient $\varphi_{1,0}$, which characterizes the heat-release rate in the liquid ($\partial l / \partial \varphi_{1,0} < 0$), and it increases with increase in the flow velocity, which determines the convective component of heat transfer through the boundary of the region occupied by the liquid ($\partial l / \partial w > 0$).

Let $\mu = \text{Re}\sigma_{11} = \text{Re}\sigma_{12} = 0.5(\varphi_{1,0} - a)$, and after substitution of $\varphi_{1,0} = 2\mu + a$ into the expression for the operator A , the operator in the neighborhood of the point $\mu = 0$ can be written as the expansion

$$A = A(\mu) = A(0) + \mu \frac{\partial A(0)}{\partial \mu}. \quad (4)$$

where

$$A(0) = \begin{vmatrix} \partial^2/\partial x^2 + a - \alpha_1 - 0.25w^2 & \varphi_{0,1} \\ -\alpha a & \partial^2/\partial x^2 - \alpha\varphi_{0,1} - 0.25w^2 \end{vmatrix}, \quad \frac{\partial A(0)}{\partial \mu} = \begin{vmatrix} 2 & 0 \\ -2\alpha & 0 \end{vmatrix}.$$

The eigenvalue $\sigma_1 = 0$ has an algebraic multiplicity of 2 and a geometrical multiplicity of 1, i.e., there is just one eigenvector

$$\mathbf{y}_1 = \begin{vmatrix} 1 \\ -\eta\varphi_{0,1}^{-1} \end{vmatrix} \sin(\lambda_1 x) \quad (\eta = \lambda_1^2 + \alpha\varphi_{0,1} + 0.25w^2)$$

that satisfies the equation $(A(0) - \sigma_1 I)\mathbf{y}_1 = 0$, which corresponds to just one eigenvector

$$\mathbf{y}_2^* = \begin{vmatrix} a\alpha \\ \eta \end{vmatrix} \sin(\lambda_1 x),$$

which is conjugate with respect to the scalar product and makes the operator $(A^*(0) - \sigma_1 I)$ vanish. Here I is a unit 2×2 matrix and

$$A^* = \left\| \begin{array}{cc} \partial^2/\partial x^2 + 2\mu + a - \alpha_1 - 0.25w^2 & -\alpha(2\mu + a) \\ \varphi_{0,1} & \partial^2/\partial x^2 - \alpha\varphi_{0,1} - 0.25w^2 \end{array} \right\|.$$

The scalar product of the vector \mathbf{y}_1 and the conjugate vector \mathbf{y}_2^* cannot be normalized since it is equal to zero by virtue of (3):

$$\begin{aligned} \langle \mathbf{y}_1, \mathbf{y}_2^* \rangle &= \int_0^l \mathbf{y}_1^t \bar{\mathbf{y}}_2^* dx = (\varphi_{0,1} a \alpha - \eta^2) \varphi_{0,1}^{-1} \int_0^l \sin^2(\lambda_1 x) dx \\ &= (\varphi_{0,1} \alpha \alpha_1 - (\lambda_1^2 + 0.25w^2)^2) \varphi_{0,1}^{-1} \int_0^l \sin^2(\lambda_1 x) dx = 0. \end{aligned}$$

(The bar indicates complex conjugacy but here $\bar{\mathbf{y}}_2^* = \mathbf{y}_2^*$ since \mathbf{y}_2^* is a real vector.)

The value of the double root is $\sigma_1 = 0$, which does not permit one to make a conclusion on the stability of the solutions of the problem (1). In this case, for a stability analysis, it is necessary to use the nonlinear terms of system (1) to eliminate the point of degeneracy of the linear problem $A(0)\mathbf{U} = 0$, defined by relations (3).

Moving from the point of degeneracy, we seek the branching eigenvalues σ_{11} and σ_{12} with perturbation of the parameter μ . Following [3], we assume that one of the eigenvalues is identically equal to zero, for example, $\sigma_{11} \equiv 0$, and the other eigenvalue σ_{12} , which is a smooth function of μ [5], determines the stability of the solution of the problem. The branches σ_{11} and σ_{12} are continuous functions of the parameter μ if the double eigenvalue corresponds to the two-dimensional zero space of the operator $A(0)$ that admits normalization.

To construct the zero space of the operator $A(0)$ with the scalar product that admits normalization, it is necessary to use the concept of generalized vectors. According to [3, 6], the vector basis of the operator $A(0)$ is determined by the following Jordan chain of equations for eigenvectors and generalized eigenvectors:

$$A(0)\mathbf{y}_1 = 0, \quad A^*(0)\mathbf{y}_2^* = 0, \quad A(0)\mathbf{y}_2 = \mathbf{y}_1, \quad A^*(0)\mathbf{y}_1^* = \mathbf{y}_2^*. \quad (5)$$

The eigenvectors \mathbf{y}_1 and \mathbf{y}_2^* are determined, and from the last two equations of (5), we obtain with accuracy up to the constant coefficients C_1 and C_2 the generalized eigenvectors

$$\mathbf{y}_2 = C_1 \left\| \begin{array}{c} 0 \\ \varphi_{0,1}^{-1} \end{array} \right\| \sin(\lambda_1 x), \quad \mathbf{y}_1^* = C_2 \left\| \begin{array}{c} 0 \\ -1 \end{array} \right\| \sin(\lambda_1 x).$$

In the constructed vector basis of the operator $A(0)$, the following relations hold: $\langle \mathbf{y}_1, \mathbf{y}_2^* \rangle = \langle A(0)\mathbf{y}_2, \mathbf{y}_2^* \rangle = \langle \mathbf{y}_2, A^*(0)\mathbf{y}_2^* \rangle = 0$, and $\langle \mathbf{y}_1, \mathbf{y}_1^* \rangle = \langle A(0)\mathbf{y}_2, \mathbf{y}_1^* \rangle = \langle \mathbf{y}_2, A^*(0)\mathbf{y}_1^* \rangle = \langle \mathbf{y}_2, \mathbf{y}_2^* \rangle$. Assuming that $C_1 = C_2 = 2\varphi_{0,1}\lambda_1/[\eta(\lambda_1 - \sin(\lambda_1 l) \cos(\lambda_1 l))]$, we obtain the normalization condition $\langle \mathbf{y}_1, \mathbf{y}_1^* \rangle = \langle \mathbf{y}_2, \mathbf{y}_2^* \rangle = 1$.

We seek solutions of the problem (1) in the form of series in exponents of the amplitude

$$\varepsilon = \langle \mathbf{U}, \mathbf{y}_1^* \rangle, \quad (6)$$

which is defined as the projection of the vector \mathbf{U} onto the eigensubspace associated with the generalized eigenvector \mathbf{y}_1^* :

$$\mathbf{U}(\varepsilon) = \sum_{n=1}^{\infty} \frac{\varepsilon^n \mathbf{U}_n}{n!}, \quad \mu(\varepsilon) = \sum_{n=1}^{\infty} \frac{\varepsilon^n \mu_n}{n!}. \quad (7)$$

The vectors $\mathbf{U}_n = \|U_{1n} \ U_{2n}\|^t$ and the coefficients μ_n are unknown and are to be determined.

Substitution of series (7) into the first equation of system (1), in which the operator A is represented as series (4), and identification of sets of terms with the same exponent of the amplitude ε lead to the system

$$A(0)\mathbf{U}_1 = 0, \quad (8)$$

$$A(0)\mathbf{U}_2 + 2\mu_1 \frac{\partial A(0)}{\partial \mu} \mathbf{U}_1 + B(\mathbf{U}_1, \mathbf{U}_1) = 0 \quad (9)$$

and equations with higher exponents of ε .

Joint solution of Eqs. (6) and (7) gives $\langle \mathbf{U}_1, \mathbf{y}_1^* \rangle = 1$, and $\langle \mathbf{U}_n, \mathbf{y}_1^* \rangle = 0$, $n \geq 2$. Hence, the eigenvalue problem (8) has the unique solution $\mathbf{U}_1 = \mathbf{y}_1$.

Equation (9) contains the unknown parameter μ_1 , the unknown vector

$$\mathbf{U}_2 = \begin{bmatrix} U_{12} & U_{22} \end{bmatrix}^t,$$

and the following vectors, which become known after solution of Eq. (8):

$$\frac{\partial A(0)}{\partial \mu} \mathbf{U}_1 = \begin{bmatrix} 2 & -2\alpha \end{bmatrix}^t \sin(\lambda_1 x),$$

$$B(\mathbf{U}_1, \mathbf{U}_1) = 8 \sin^2(\lambda_1 x) \exp(-0.5wx) \begin{bmatrix} \varphi_{2,0}^0 - \eta\varphi_{1,1}^0 + \eta^2\varphi_{0,2}^0 \\ -\alpha(\varphi_{2,0}^0 - \eta\varphi_{1,1}^0 + \eta^2\varphi_{0,2}^0) \end{bmatrix}.$$

Here $\varphi_{i,j}^0 = \varphi_{i,j}(x=0)$.

Calculating the scalar product of Eq. (9) into the vector \mathbf{y}_2^* and using the orthogonality condition $\langle A(0)\mathbf{U}_2, \mathbf{y}_2^* \rangle = 0$, we have the parameter

$$\mu_1 = 0.5 \langle B(\mathbf{U}_1, \mathbf{U}_1), \mathbf{y}_2^* \rangle \left\langle \frac{\partial A(0)}{\partial \mu} \mathbf{U}_1, \mathbf{y}_2^* \right\rangle^{-1}. \quad (10)$$

Substitution of the expressions

$$\langle B(\mathbf{U}_1, \mathbf{U}_1), \mathbf{y}_2^* \rangle = \int_0^l B(\mathbf{U}_1, \mathbf{U}_1)^t \mathbf{y}_2^* dx = 8C_1\alpha\Delta(a-\eta) \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx,$$

$$\left\langle \frac{\partial A(0)}{\partial \mu} \mathbf{U}_1, \mathbf{y}_2^* \right\rangle = \int_0^l \left[\frac{\partial A(0)}{\partial \mu} \mathbf{U}_1 \right]^t \mathbf{y}_2^* dx = 2C_1\alpha(a-\eta) \int_0^l \sin^2(\lambda_1 x) dx$$

($\Delta = \varphi_{2,0}^0 - \eta\varphi_{1,1}^0 + \eta^2\varphi_{0,2}^0$) into Eq. (10) gives

$$\mu_1 = 4\Delta \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx \left[\int_0^l \sin^2(\lambda_1 x) dx \right]^{-1}. \quad (11)$$

The integrals on the right side of (11) are expressed in terms of elementary functions.

According to [3], the definition of the stability of the stationary bifurcation solution $A\mathbf{U} + B(\mathbf{U}, \mathbf{U}) + C(\mathbf{U}, \mathbf{U}, \mathbf{U}) = 0$ reduces to analysis of the spectral problem

$$\gamma\tilde{\mathbf{U}} = A\tilde{\mathbf{U}} + B(\mathbf{U}(\varepsilon), \tilde{\mathbf{U}}) + C(\mathbf{U}(\varepsilon), \mathbf{U}(\varepsilon), \tilde{\mathbf{U}}) \quad (12)$$

for small perturbations $\exp(\gamma t)\tilde{\mathbf{U}}$ of the solution $\mathbf{U}(\varepsilon)$. Substituting series (7) into (12), we obtain

$$\gamma\tilde{\mathbf{U}} = A(0)\tilde{\mathbf{U}} + \varepsilon \left[\mu_1 \frac{\partial A(0)}{\partial \mu} \tilde{\mathbf{U}} + B(\mathbf{U}_1, \tilde{\mathbf{U}}) \right] + O(\varepsilon^2\tilde{\mathbf{U}}). \quad (13)$$

The unperturbed solution of the problem $\tilde{\mathbf{U}}$ can be decomposed into a part $\tilde{\mathbf{U}}_1$ that belongs to the zero space of the operator $A(0)$ and an additional small part $\tilde{\mathbf{U}}_2$ with zero projection $\langle \tilde{\mathbf{U}}_2, \mathbf{y}_i^* \rangle = 0$, where $i = 1$ and 2 .

The part of the solution that belongs to the zero space of the operator $A(0)$ is a linear combination of the vectors $\mathbf{y}_1, \mathbf{y}_2$: $\tilde{\mathbf{U}}_1 = \theta_1\mathbf{y}_1 + \theta_2\mathbf{y}_2$, and the general solution of the unperturbed problem has the form

$$\tilde{\mathbf{U}} = \theta_1\mathbf{y}_1 + \theta_2\mathbf{y}_2 + \tilde{\mathbf{U}}_2. \quad (14)$$

Substitution of the unperturbed solution (14) into Eq. (13) gives

$$\gamma \tilde{U}_1 = A(0)\tilde{U}_1 + \varepsilon \left[\mu_1 \frac{\partial A(0)}{\partial \mu} \tilde{U}_1 + B(\mathbf{y}_1, \tilde{U}_1) \right] + O(\varepsilon \tilde{U}_2) + O(\varepsilon^2 \tilde{U}), \quad (15)$$

where

$$A(0)\tilde{U}_1 = \theta_2 \mathbf{y}_1; \quad \frac{\partial A(0)}{\partial \mu} \tilde{U}_1 = 2\theta_1 \sin(\lambda_1 x) \|1 - \alpha\|^t;$$

$$B(\mathbf{y}_1, \tilde{U}_1) = (2\eta\theta_1 - C_1\theta_2)\varphi_{0,1}^{-1}\varphi_{1,1}^0 \sin^2(\lambda_1 x) \exp(-0.5wx) \| -1 - \alpha \|^t.$$

Joss and Joseph [3] proved that $O(\varepsilon \tilde{U}_2) = O(\varepsilon^2 \tilde{U})$, and, hence, the last two terms in (15) have the same order of smallness.

Scalar multiplication of the left and right sides of Eq. (15) by the vectors \mathbf{y}_1^* and \mathbf{y}_2^* leads to the following system of algebraic equations for the unknown constants θ_1 and θ_2 :

$$\gamma \langle \tilde{U}_1, \mathbf{y}_i^* \rangle = \langle A(0)\tilde{U}_1, \mathbf{y}_i^* \rangle + \varepsilon \left[\mu_1 \left\langle \frac{\partial A(0)}{\partial \mu} \tilde{U}_1, \mathbf{y}_i^* \right\rangle + \langle B(\mathbf{y}_1, \tilde{U}_1), \mathbf{y}_i^* \rangle \right] + O(\varepsilon^2 \tilde{U}), \quad i = 1, 2. \quad (16)$$

To solve system (16), we calculate the scalar products

$$\langle \tilde{U}_1, \mathbf{y}_1^* \rangle = \theta_1, \quad \langle \tilde{U}_1, \mathbf{y}_2^* \rangle = \theta_2, \quad \langle A(0)\tilde{U}_1, \mathbf{y}_1^* \rangle = \theta_2, \quad \langle A(0)\tilde{U}_1, \mathbf{y}_2^* \rangle = 0,$$

$$\left\langle \frac{\partial A(0)}{\partial \mu} \tilde{U}_1, \mathbf{y}_1^* \right\rangle = 2C_2\alpha\theta_1 \int_0^l \sin^2(\lambda_1 x) dx,$$

$$\left\langle \frac{\partial A(0)}{\partial \mu} \tilde{U}_1, \mathbf{y}_2^* \right\rangle = 2\alpha(\alpha - \eta)\theta_1 \int_0^l \sin^2(\lambda_1 x) dx,$$

$$\langle B(\mathbf{y}_1, \tilde{U}_1), \mathbf{y}_1^* \rangle = \alpha C_2(C_1\theta_2 - 2\eta\theta_1)\varphi_{0,1}^{-1}\varphi_{1,1}^0 \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx,$$

$$\langle B(\mathbf{y}_1, \tilde{U}_1), \mathbf{y}_2^* \rangle = \alpha(a - \eta)(C_1\theta_2 - 2\eta\theta_1)\varphi_{0,1}^{-1}\varphi_{1,1}^0 \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx$$

and determine the coefficients

$$e_1 = 2C_2\alpha\mu_1 \int_0^l \sin^2(\lambda_1 x) dx - 2C_2\alpha\eta\varphi_{0,1}^{-1}\varphi_{1,1}^0 \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx,$$

$$e_2 = C_1C_2\alpha\varphi_{0,1}^{-1}\varphi_{1,1}^0 \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx,$$

$$e_3 = 2\alpha(a - \eta)\mu_1 \int_0^l \sin^2(\lambda_1 x) dx - 2\alpha(a - \eta)\eta\varphi_{0,1}^{-1}\varphi_{1,1}^0 \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx,$$

$$e_4 = C_1\alpha(a - \eta)\varphi_{0,1}^{-1}\varphi_{1,1}^0 \int_0^l \sin^3(\lambda_1 x) \exp(-0.5wx) dx.$$

System (16) then becomes

$$\gamma\theta_1 = e_1\varepsilon\theta_1 + (1 + e_2\varepsilon)\theta_2 + O(\varepsilon^2), \quad \gamma\theta_2 = e_3\varepsilon\theta_1 + e_4\varepsilon\theta_2 + O(\varepsilon^2). \quad (17)$$

System (17) has a nontrivial solution for θ_1 and θ_2 if

$$\det \begin{vmatrix} e_1\varepsilon - \gamma + O(\varepsilon^2) & 1 + e_2\varepsilon + O(\varepsilon^2) \\ e_3\varepsilon + O(\varepsilon^2) & e_4\varepsilon - \gamma + O(\varepsilon^2) \end{vmatrix} = 0. \quad (18)$$

It is obvious that γ is an eigenvalue of the matrix

$$\begin{vmatrix} e_1\varepsilon + O(\varepsilon^2) & 1 + e_2\varepsilon + O(\varepsilon^2) \\ e_3\varepsilon + O(\varepsilon^2) & e_4\varepsilon + O(\varepsilon^2) \end{vmatrix} = 0.$$

Taking into account that $\sqrt{e_3\varepsilon + O(\varepsilon^2)} = \sqrt{e_3\varepsilon} + O(\varepsilon^{3/2})$, we find roots of the quadratic equation (18)

$$\gamma_{1,2} = \pm\sqrt{e_3\varepsilon} + 0.5(e_1 + e_4)\varepsilon + O(\varepsilon^{3/2}). \quad (19)$$

The expression for the eigenvalues (19) corresponds (with accuracy to notation) to the formula obtained within the framework of the general theory [3]. If the real parts of the roots γ_1 and γ_2 have the same signs, the bifurcation solution is stable on one side of the critical point and unstable on the other; when the signs of the real parts γ_1 and γ_2 are opposite, the solution is unstable on both sides of the critical value. Finally, if the roots of Eq. (19) are equal to zero, the stability problem is solved by considering the higher-order terms $C(\mathbf{U}, \mathbf{U}, \mathbf{U})$ of Eq. (1).

As mentioned above, to determine the stability of the solution of the bifurcation problem (1) for the parameter values corresponding to the line of intersection of surfaces I and II (see Fig. 1), it is necessary to consider the effect of the nonlinear terms. Let the coefficients at the nonlinear terms be $\varphi_{2,0} = 0.2$, $\varphi_{1,1} = 0.2$, and $\varphi_{0,2} = 0$. We determine the roots of Eq. (19) for an arbitrary point on the line of intersection of surfaces I and II, for example, the point $\varphi_{1,0} = 0.4$, and $w = 0.25$. From calculations, we obtain

$$\begin{aligned} \lambda_1 &= 0.29, \quad a = 0.4, \quad \eta = 0.2, \quad l = 4.009, \quad \mathbf{y}_1 = \|1 \quad -2\|^t \sin(0.29x), \\ \mathbf{y}_1^* &= \|0 \quad -1\|^t \sin(0.29x), \quad \mathbf{y}_2 = \|0 \quad 10\|^t \sin(0.29x), \\ \mathbf{y}_2^* &= \|0.4 \quad 0.2\|^t \sin(0.29x), \quad C_1 = C_2 = 0.317, \quad e_1 = 0.119, \quad e_2 = 0.18, \\ e_3 &= 0.065, \quad e_4 = 0.1, \quad \gamma_1 = 0.364, \quad \gamma_2 = -0.146. \end{aligned}$$

Hence it follows that one of the real roots γ_1 and γ_2 is positive for any sign of μ and the bifurcation solution at the selected point is unstable on both sides of the critical value.

We note that when the right side of the differential equation (1) contains the defect $\Phi(0,0) \neq 0$, the bifurcation solution is split into separate solutions. The general method of [3] for solving problems with perturbations due to defects does not depend on the procedure of constructing the zero space of the generating operator. Hence, for $\langle \Phi(0,0), \mathbf{y}_i \rangle \neq 0$ ($i = 1$ and 2) separate solutions are obtained by the same procedure as in the case of generating operators with a simple structure [1].

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